ASYMPTOTIC FORM OF THE AXISYMMETRIC ELASTICITY PROBLEM FOR AN ANISOTROPIC CYLINDRICAL SHELL

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The idea of an asymptotic analysis of elasticity problems for thin bodies is due to Friedrichs [1, 2], who showed that the solution of the linear elasticity problem for an isotropic plate differs from the solution of the classical Kirchhoff equations in a boundary layer localized at the edge of the plate.

An asymptotic analysis of elasticity problems of isotropic and anisotropic shells was made in [3-9]. These papers combined the use of the equations of linear elasticity theory, a choice of the degree of stretching of variables from the feasibility condition for passing to the Kirchhoff–Love equations as a limit, and the expansion of the required solution in powers of a small parameter proportional to the thickness.

An exact asymptotic analysis of the axisymmetric elasticity problem for an isotropic cylindrical shell was performed in [10]. The method of homogeneous solutions was used to estimate the asymptotic orders of the characteristic roots of the system and to establish the existence of three corresponding particular solutions with different rates of change along the axial coordinate.

By using a simpler method than employed in [10] we establish the same results for an anisotropic cylindrical shell as was obtained there for an isotropic shell: the asymptotic elasticity problem has a regular solution and not more than two boundary-layer solutions with different variability indices. We investigate the asymptotic form of the problem and construct iteration systems of equations for determining solutions of all three types.

1. We consider the linear problem of the axisymmetric static deformation of an orthotropic cylindrical shell as a three-dimensional elastic body.

Let a, b, and c be, respectively, the half-length, radius of the middle surface, and half-thickness of the shell; t_n is an orthogonal coordinate system fixed on the middle surface; the h_n are its metric Lamé coefficients; e_{mn}, e_{44}, e_{55}, and e_{66} are elements of the symmetric compliance matrix of the orthotropic material; the $P_n = bp_n$ are the components of the external body forces; the σ_{mn} are the components of the symmetric stress tensor; the ε_{mn} are the components of the symmetric strain tensor; the $u_m = bw_n$ are the components of the displacement vector (the subscripts m and n take on the values 1, 2, 3).

The independent variables range over the limits

$$t_1 \in [-\alpha, \alpha],\ t_2 \in [-\pi, \pi],\ t_3 \in [-1, 1]\ (\alpha = a/b).$$

The Lamé coefficients are given by the equations

$$h_1 = b,\ h_2 = b + ct_3,\ h_3 = c.$$

The compliance coefficients do not depend on ε ($\varepsilon = c/b$). From the condition of axial symmetry the variable parameters introduced do not depend on the tangential coordinate t_2. Then the static problem of the linear elasticity theory of an anisotropic elastic body is formulated by the following system of differential equations [11]:

$$\varepsilon_{11} = \partial w_1/\partial t_1 = e_{11}\sigma_{11} + e_{12}\sigma_{22} + e_{13}\sigma_{33},\ \varepsilon_{13} = \varepsilon\partial w_3/\partial t_1 + \partial w_1/\partial t_3 = \varepsilon e_{44}\sigma_{13}, \tag{1.1}$$

$$a_2\varepsilon_{22} = w_3 = a_2(e_{21}\sigma_{11} + e_{22}\sigma_{22} + e_{23}\sigma_{33}),$$

$$\varepsilon_{23} = \frac{\partial w_2}{\partial t_3} - \varepsilon\frac{w_2}{a_2} = \varepsilon e_{55}\sigma_{23},\quad \varepsilon_{33} = \frac{\partial w_3}{\partial t_3} = \varepsilon(e_{31}\sigma_{11} + e_{32}\sigma_{22} + e_{33}\sigma_{33}),$$

$$\varepsilon_{12} = \frac{\partial w_2}{\partial t_1} = e_{66}\sigma_{12},\quad \varepsilon\frac{\partial}{\partial t_1}(a_2\sigma_{11}) + \frac{\partial}{\partial t_3}(a_2\sigma_{13}) + \varepsilon a_2 p_1 = 0,$$

$$\varepsilon\frac{\partial}{\partial t_1}(a_2\sigma_{12}) + \frac{\partial}{\partial t_3}(a_2\sigma_{23}) + \varepsilon\sigma_{23} + \varepsilon a_2 p_2 = 0,$$

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$$\varepsilon \frac{\partial}{\partial t_1}(a_2\sigma_{13}) + \frac{\partial}{\partial t_3}(a_2\sigma_{33}) - \varepsilon\sigma_{22} + \varepsilon a_2 p_3 = 0,$$
$$a_2 = 1 + \varepsilon t_3.$$

Equations (1.1) are supplemented by one of the following systems of boundary conditions:

a) a system of stresses

$$\sigma_{n3}|_{t_3=\mp 1} = S_{n3}^{\mp}(t_1), \quad \sigma_{n1}|_{t_1=\mp\alpha} = S_{n1}^{\mp}(t_3);$$

b) a system of displacements

$$w_n|_{t_3=\mp 1} = H_{n3}^{\mp}(t_1), \quad w_n|_{t_1=\mp\alpha} = H_{n1}^{\mp}(t_3); \tag{1.2}$$

c) a mixed system which combines some of conditions a) and some of b).

From the definition of a shell

$$\varepsilon \ll 1, \ \varepsilon \ll \alpha. \tag{1.3}$$

Consequently, system (1.1) contains a small parameter. Since for $\varepsilon = 0$ the derivatives of the four required functions w_3, σ_{11}, σ_{12}, and σ_{13} with respect to t_1 in (1.1) vanish, the system is singularly degenerate (singularly perturbed) [12]. It is characteristic of such a system that its solution does not converge to the degenerate form (for $\varepsilon = 0$) in the immediate neighborhood of bounding values of that variable with respect to which the degeneracy occurs. The asymptotic method of boundary functions formulated in [12] for systems of ordinary differential equations takes account of this characteristic of singularly perturbed systems. The application of this method to boundary value problem (1.1), (1.2) requires preliminary analysis.

System (1.1) corresponds to a homogeneous system which admits particular solutions of the form

$$w_n = w_n^0 \exp(t_1/\lambda + t_3/\mu), \quad \sigma_{mn} = \sigma_{mn}^0 \exp(t_1/\lambda + t_3/\mu),$$

where w_n^0, σ_{mn}^0, λ, and μ are constants. The corresponding characteristic equation determines the following relations between its roots λ and μ and the small parameter ε: λ and μ do not depend on ε, $\lambda \sim \varepsilon\mu$, and $\lambda \sim \sqrt{\varepsilon\mu}$; that is, the system has three types of solutions: 1) a regular (internal) solution with a variability index equal to zero; 2) a boundary-layer solution with a variability index equal to $1/2$; 3) a boundary-layer solution with a variability index equal to 1. The variability index is defined as the order of λ relative to ε [7]. These solutions are constructed by using three asymptotic expansions: a regular expansion in powers of ε without stretching of the axial coordinate; a boundary expansion in powers of ε with stretching of the axial coordinate proportional to $\varepsilon^{1/2}$; a boundary expansion in powers of ε with stretching of the axial coordinate proportional to ε. Consequently, the required functions w_n and σ_{mm} can be written as the sums

$$\sigma_{mn} = X_{mn} + Y_{mn} + Z_{mn}, \quad w_n = U_n + V_n + W_n, \quad \varepsilon_{mn} = U_{mn} + V_{mn} + W_{mn}, \tag{1.4}$$

in which X_{mn}, U_n, and U_{mn} are regular series with a zero variability index; Y_{mn}, V_n, and V_{mn} are boundary series with a variability index of $1/2$; Z_{mn}, W_n, and W_{mn} are boundary series with a variability index of 1.

2. The regular expansion is represented by the series

$$U_n(t_1, t_3) = \sum_{k=0}^{\infty} \varepsilon^k u_n^{(k)}(t_1, t_3), \quad X_{mn}(t_1, t_3) = \sum_{k=0}^{\infty} \varepsilon^k x_{mn}^{(k)}(t_1, t_3), \tag{2.1}$$

$$X_{n3}(t_1, t_3) = \varepsilon \sum_{k=0}^{\infty} \varepsilon^k x_{n3}^{(k)}(t_1, t_3), \quad U_{mn}(t_1, t_3) = \sum_{k=0}^{\infty} \varepsilon^k u_{mn}^{(k)}(t_1, t_3).$$

Substitution of these series into (1.1) and a comparison of coefficients of identical powers of ε leads to a sequence of systems, the first of which (corresponding to coefficients of ε^0) has the form

$$u_{11}^{(0)} = \frac{\partial u_1^{(0)}}{\partial t_1} = e_{11}x_{11}^{(0)} + e_{12}x_{22}^{(0)}, \quad u_{13}^{(0)} = \frac{\partial u_1^{(0)}}{\partial t_3} = 0, \tag{2.2}$$

$$u_{22}^{(0)} = u_3^{(0)} = e_{21}x_{11}^{(0)} + e_{22}x_{22}^{(0)}, \quad u_{23}^{(0)} = \frac{\partial u_2^{(0)}}{\partial t_3} = 0,$$

$$u_{33}^{(0)} = \frac{\partial u_3^{(0)}}{\partial t_3} = 0, \quad u_{12}^{(0)} = \frac{\partial u_2^{(0)}}{\partial t_1} = e_{66}x_{12}^{(0)};$$

$$\frac{\partial}{\partial t_1}(a_2 x_{11}^{(0)}) + \frac{\partial}{\partial t_3}(a_2 x_{13}^{(0)}) + a_2 p_1 = 0; \tag{2.2a}$$

$$\frac{\partial}{\partial t_1}\left(a_2 x_{12}^{(0)}\right) + \frac{\partial}{\partial t_3}\left(a_2 x_{23}^{(0)}\right) + a_2 p_2 = 0; \tag{2.2b}$$

$$\frac{\partial}{\partial t_3}\left(a_2 x_{33}^{(0)}\right) - x_{22}^{(0)} + a_2 p_3 = 0. \tag{2.2c}$$

This system determines the displacements as functions of the single coordinate t_1, and leads to the closed system of the zero-moment theory of shells

$$\begin{gathered} u_1^{(0)} \equiv u_{10}(t_1), \quad u_2^{(0)} \equiv u_{20}(t_1), \quad u_3^{(0)} \equiv u_{30}(t_1), \\ e_{11}X_{11}^{(0)} + e_{12}X_{22}^{(0)} = 2\frac{du_1^{(0)}}{dt_1}, \\ e_{21}X_{11}^{(0)} + e_{22}X_{22}^{(0)} = 2u_3^{(0)}, \quad e_{66}X_{12}^{(0)} = 2\frac{du_2}{dt_1}, \\ X_{11,1}^{(0)} + f_1^{(0)} = 0, \quad X_{11}^{(0)} = \int_{-1}^{1} a_2 x_{11}^{(0)} dt_3, \quad f_1^{(0)} = \left[a_2 x_{13}^{(0)}\right]_{-1}^{1} + \int_{-1}^{1} a_2 p_1 dt_3, \\ X_{12,1}^{(0)} + f_2^{(0)} = 0, \quad X_{12}^{(0)} = \int_{-1}^{1} a_2 x_{12}^{(0)} dt_3, \quad f_2^{(0)} = \left[a_2 x_{23}^{(0)}\right]_{-1}^{1} + \int_{-1}^{1} a_2 p_2 dt_3, \\ X_{22}^{(0)} - f_3^{(0)} = 0, \quad X_{22}^{(0)} = \int_{-1}^{1} x_{22}^{(0)} dt_3, \quad f_3^{(0)} = \left[a_2 x_{33}^{(0)}\right]_{-1}^{1} + \int_{-1}^{1} a_2 p_3 dt_3 \end{gathered} \tag{2.3}$$

with the supplementary Eqs. (2.2a)-(2.2c), which determine the transverse stresses in terms of the stresses of the zero-moment state. The transverse strains are determined in terms of the known stresses from the following generalized Hooke's law equations:

$$\begin{gathered} U_{13}^{0} = \varepsilon e_{44} x_{13}^{(0)}, \quad U_{23}^{0} = \varepsilon e_{55} x_{23}^{(0)}, \\ U_{33}^{0} = \varepsilon\left(e_{31}x_{11}^{(0)} + e_{32}x_{22}^{(0)} + e_{33}x_{33}^{(0)}\right). \end{gathered}$$

Expansion (2.1) can satisfy all the boundary conditions in both stresses and displacements on the cylindrical surfaces of the shell, but cannot satisfy the boundary conditions on the ends.

3. The first boundary expansion is performed in the homogeneous system corresponding to (1.1), after replacing the variable t_1 by

$$\vartheta_1 = t_1/\sqrt{\varepsilon}.$$

It is represented by the series

$$\begin{gathered} Y_{ij}(\vartheta_1, t_3) = \sum_{k=0}^{\infty} \varepsilon^{k/2} y_{ij}^{(k)}(\vartheta_1, t_3), \\ Y_{i3}(\vartheta_1, t_3) = \varepsilon^{1/2}\sum_{k=0}^{\infty} \varepsilon^{k/2} y_{i3}^{(k)}(\vartheta_1, t_3), \quad Y_{33}(\vartheta_1, t_3) = \varepsilon\sum_{k=0}^{\infty} \varepsilon^{k/2} y_{33}^{(k)}(\vartheta_1, t_3), \\ V_i(\vartheta_1, t_3) = \varepsilon^{1/2}\sum_{k=0}^{\infty} \varepsilon^{k/2} v_i^{(k)}(\vartheta_1, t_3), \quad V_3(\vartheta_1, t_3) = \sum_{k=0}^{\infty} \varepsilon^{k/2} v_3^{(k)}(\vartheta_1, t_3), \\ V_{mn}(\vartheta_1, t_3) = \sum_{k=0}^{\infty} \varepsilon^{k/2} v_{mn}^{(k)}(\vartheta_1, t_3). \quad i, j = 1, 2. \end{gathered} \tag{3.1}$$

The agreement of the dependent variables indicated here leads to the elimination of fractional powers of the parameter ε from the original system. Consequently, its solution can be represented by an expansion in integral powers of ε, so that the summation in series (3.1) needs to be performed only over even values of k.

By substituting series (3.1) into system (1.1) with the "stretched" independent variable ϑ_1, and equating coefficients of identical integral powers of ε, a sequence of systems is obtained, the first of which (corresponding to coefficients of ε^0) has the form

$$\begin{gathered} v_{11}^{(0)} = \frac{\partial v_1^{(0)}}{\partial \vartheta_1} = e_{11}y_{11}^{(0)} + e_{12}y_{22}^{(0)}, \quad v_{13}^{(0)} = \frac{\partial v_3^{(0)}}{\partial \vartheta_1} + \frac{\partial v_1^{(0)}}{\partial t_3} = 0, \\ v_{22}^{(0)} = v_3^{(0)} = e_{21}y_{11}^{(0)} + e_{22}y_{22}^{(0)}, \quad v_{23}^{(0)} = \frac{\partial v_2^{(0)}}{\partial t_3} = 0, \\ v_{33}^{(0)} = \frac{\partial v_3^{(0)}}{\partial t_3} = 0, \quad v_{12}^{(0)} = \frac{\partial v_2^{(0)}}{\partial \vartheta_1} = e_{66}y_{12}^{(0)}; \end{gathered} \tag{3.2}$$

$$\frac{\partial}{\partial \vartheta_1}\left(a_2 y_{11}^{(0)}\right)+\frac{\partial}{\partial t_3}\left(a_2 y_{13}^{(0)}\right)=0; \quad (3.2a)$$

$$\frac{\partial}{\partial \vartheta_1}\left(a_2 y_{12}^{(0)}\right)+\frac{\partial}{\partial t_3}\left(a_2 y_{23}^{(0)}\right)=0; \quad (3.2b)$$

$$\frac{\partial}{\partial \vartheta_1}\left(a_2 y_{13}^{(0)}\right)+\frac{\partial}{\partial t_3}\left(a_2 y_{33}^{(0)}\right)-y_{22}^{(0)}=0. \quad (3.2c)$$

This system reduces to the closed system of Kirchhoff–Love equations

$$v_1^{(0)}\equiv v_{10}(\vartheta_1)-t_3\frac{dv_{30}}{d\vartheta_1},\quad v_2^{(0)}\equiv v_{20}(\vartheta_1),\quad v_3^{(0)}\equiv v_{30}(\vartheta_1), \quad (3.3)$$

$$Y_{11}^{(0)}=\frac{2}{\Omega}\left(e_{22}\frac{dv_{10}(\vartheta_1)}{d\vartheta_1}-e_{12}v_{30}(\vartheta_1)\right),\quad \Omega=e_{11}e_{22}-e_{12}^2,$$

$$Y_{12}^{(0)}=\frac{2}{e_{66}}\frac{dv_{20}(\vartheta_1)}{d\vartheta_1},\quad Y_{22}^{(0)}=\frac{2}{\Omega}\left(e_{11}v_{30}(\vartheta_1)-e_{12}\frac{dv_{10}(\vartheta_1)}{d\vartheta_1}\right),$$

$$Y_{11,1}^{(0)}+g_1^{(0)}=0,\quad Y_{11}^{(0)}=\int_{-1}^{1}a_2y_{11}^{(0)}dt_3,\quad g_1^{(0)}=\left[a_2y_{13}^{(0)}\right]_{-1}^{1},$$

$$Y_{12,1}^{(0)}+g_2^{(0)}=0,\quad Y_{12}^{(0)}=\int_{-1}^{1}a_2y_{12}^{(0)}dt_3,\quad g_2^{(0)}=\left[a_2y_{23}^{(0)}\right]_{-1}^{1},$$

$$Y_{13,1}^{(0)}-Y_{22}^{(0)}+g_3^{(0)}=0,\quad Y_{22}^{(0)}=\int_{-1}^{1}y_{22}^{(0)}dt_3,\quad Y_{13}^{(0)}=\int_{-1}^{1}a_2y_{13}^{(0)}dt_3,$$

$$M_{11,1}^{(0)}-Y_{13}^{(0)}=0,\quad M_{11}^{(0)}=\int_{-1}^{1}t_3a_2y_{11}^{(0)}dt_3,\quad g_3^{(0)}=\left[a_2y_{33}^{(0)}\right]_{-1}^{1},$$

$$M_{11}^{(0)}=-\frac{2}{3}\frac{1}{\Omega}\frac{d^2v_{30}(\vartheta_1)}{d\vartheta_1^2}$$

with the supplementary equations (3.2a)-(3.2c) which determine the transverse stresses in terms of the tangential stresses. The transverse strains are determined from the following generalized Hooke's law equations:

$$V_{13}^0=\varepsilon e_{44}y_{13}^{(0)},\quad V_{23}^0=\varepsilon e_{55}y_{23}^{(0)},$$

$$V_{33}^0=\varepsilon\left(e_{31}y_{11}^{(0)}+e_{32}y_{22}^{(0)}+e_{33}y_{33}^{(0)}\right).$$

Expansion (3.1) gives a solution of the boundary-layer type with a variability index of 1/2, which still does not permit satisfying all the boundary conditions on the ends of the shell.

4. The secondary boundary expansion is performed in the homogeneous system corresponding to (1.1), after replacing the variable t_1 by

$$\tau_1=t_1/\varepsilon.$$

It is represented by the series

$$Z_{mn}(\tau_1,t_3)=\sum_{k=0}^{\infty}\varepsilon^k z_{mn}^{(k)}(\tau_1,t_3), \quad (4.1)$$

$$W_n(\tau_1,t_3)=\varepsilon\sum_{k=0}^{\infty}\varepsilon^k w_n^{(k)}(\tau_1,t_3),$$

$$W_{mn}(\tau_1,t_3)=\sum_{k=0}^{\infty}\varepsilon^k w_{mn}^{(k)}(\tau_1,t_3)$$

and leads to a sequence of systems, the first of which (for ε^0) has the form

$$w_{11}^{(0)}=\frac{\partial w_1^{(0)}}{\partial\tau_1}=e_{11}z_{11}^{(0)}+e_{12}z_{22}^{(0)}+e_{13}z_{33}^{(0)}, \quad (4.2)$$

$$w_{13}^{(0)}=\frac{\partial w_3^{(0)}}{\partial\tau_1}+\frac{\partial w_1^{(0)}}{\partial t_3}=e_{44}z_{13}^{(0)},$$

$$w_{22}^{(0)}=e_{21}z_{11}^{(0)}+e_{22}z_{22}^{(0)}+e_{23}z_{33}^{(0)}=0,\quad w_{23}^{(0)}=\frac{\partial w_2^{(0)}}{\partial t_3}=e_{55}z_{23}^{(0)},$$

$$w_{33}^{(0)}=\frac{\partial w_3^{(0)}}{\partial t_3}=e_{31}z_{11}^{(0)}+e_{32}z_{22}^{(0)}+e_{33}z_{33}^{(0)},\quad w_{12}^{(0)}=\frac{\partial w_2^{(0)}}{\partial\tau_1}=e_{66}z_{12}^{(0)},$$

$$\frac{\partial z_{11}^{(0)}}{\partial \tau_1} + \frac{\partial z_{13}^{(0)}}{\partial t_3} = 0, \quad \frac{\partial z_{12}^{(0)}}{\partial \tau_1} + \frac{\partial z_{23}^{(0)}}{\partial t_3} = 0, \quad \frac{\partial z_{13}^{(0)}}{\partial \tau_1} + \frac{\partial z_{33}^{(0)}}{\partial t_3} = 0.$$

Expansion (4.1) gives a solution of the boundary-layer type with a variability index 1 (Friedrichs boundary layer) which in each approximation permits the removal of the discrepancies in boundary conditions on the ends of the cylinder generated by the first two solutions.

5. The question of the convergence of the asymptotic series constructed requires special study. Favorable results were obtained in [13, 14] for plates.

The ellipticity of system (4.2) and the possibility of satisfying all boundary conditions in each approximation ensure the absence of angular boundary layers [15].

When series (1.4), (2.1), (3.1), and (4.1) converge they give an exact solution of the original problem. Their partial sums give an asymptotically exact solution.

In particular, the functions

$$w_n = U_n^0 + V_n^0 + W_n^0 \text{ and } \sigma_{mn} = X_{mn}^0 + Y_{mn}^0 + Z_{mn}^0,$$

where U_n^0, X_{mn}^0; V_n^0, Y_{mn}^0; W_n^0, Z_{mn}^0 are, respectively, the zero-approximation solutions of systems (2.2), (3.2), and (4.2), form the simplest asymptotically exact solution of the original problem which satisfies all the boundary conditions.

The functions $w_n = U_n^0 + V_n^0$, $\sigma_{mn} = X_{mn}^0 + Y_{mn}^0$ form a solution which is asymptotically exact outside small (length of the order $b\varepsilon$) boundary zones of the shell. This solution is determined either directly by integrating systems (2.2) and (3.2), or by successive integrations of "shell" systems (2.3) and (3.3) with "supplementary" equations (2.2a)-(2.2c) and (3.2a)-(3.2c).

The functions $w_n = U_n^0$ and $\sigma_{mn} = X_{mn}^0$ are a solution which is asymptotically exact outside boundary zones of the shell of length of the order $b\sqrt{\varepsilon}$, and are determined either by direct integration of system (2.2) or by successive integration of (2.3) with the supplementary Eqs. (2.2a)-(2.2c). Thus, the asymptotic method gives a theoretical basis for so-called iterative theories of shells [11]. It admits iterations refining the Kirchhoff–Love theory and iterations refining the zero-moment theory of shells.

For an anisotropic shell which has reduced resistance to transverse stresses, for example a shell reinforced with rigid fibers parallel to the middle surface, the coefficients e_{33}, e_{44}, and e_{55} in Eqs. (1.1) are substantially larger than the other compliance coefficients. In this case the necessary condition for the solution to be represented by the series (1.4), (2.1), (3.1), and (4.1) is

$$e\varepsilon \ll 1,$$

where $e = \max(e_{33}, e_{44}, e_{55})$, which narrows down the domain of definition of the dimensions of the shell in comparison with (1.3). At the same time the contribution of transverse stresses and strains to the total stress–strain state of the shell is increased. It becomes increasingly necessary to use the "supplementary" equations (2.2a)-(2.2c) and (3.2a)-(3.2c) in determining the transverse components of the stress and strain tensors in terms of the tangential components in the first approximation.

Thus, the solution of an axisymmetric elasticity problem has been constructed for an orthotropic cylindrical shell. An analysis of the characteristic roots of system (1.1) established that an axisymmetric elasticity problem for an orthotropic cylindrical shell has a regular (internal) solution and two boundary-layer solutions with significantly different variability indices: one with a variability index of $^1/_2$ (Kirchhoff–Love boundary layer) and one with a variability index of 1 (Friedrichs boundary layer). Iteration systems of equations have been constructed for determining all three types of solutions.

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